Solutions - Midterm Exam

(October 19th @ 3:30 pm)

Presentation and clarity are very important! Show your procedure!

PROBLEM 1 (22 PTS)

a) Complete the following table. The decimal numbers are unsigned: (3 pts.)

Decimal	BCD	Binary	Reflective Gray Code
51	01010001	110011	101010
69	01101001	01000101	01100111

b) Complete the following table. Use the fewest number of bits in each case: (15 pts.)

	REPRESENTATION		
Decimal	Sign-and-magnitude	1's complement	2's complement
-19	110011	101100	101101
0	00	11111	0
-8	11000	10111	1000
29	011101	011101	011101
-16	110000	101111	10000
-24	111000	100111	101000

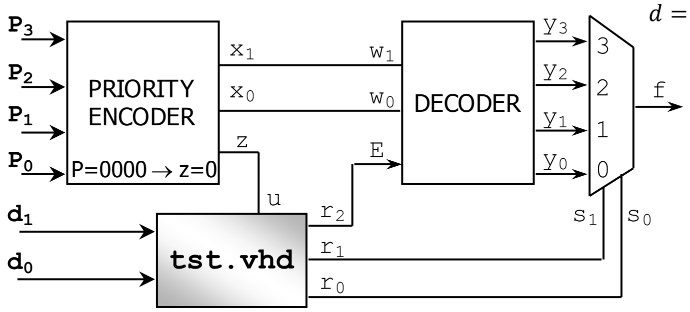
c) Convert the following decimal numbers to their 2's complement representations. (4 pts.)

- ✓ -16.5
  +16.5 = 010000.1 ⇒ -16.3125 = 101111.1
- ✓ 18.75
  +18.75 = 010010.11

PROBLEM 2 (14 PTS)

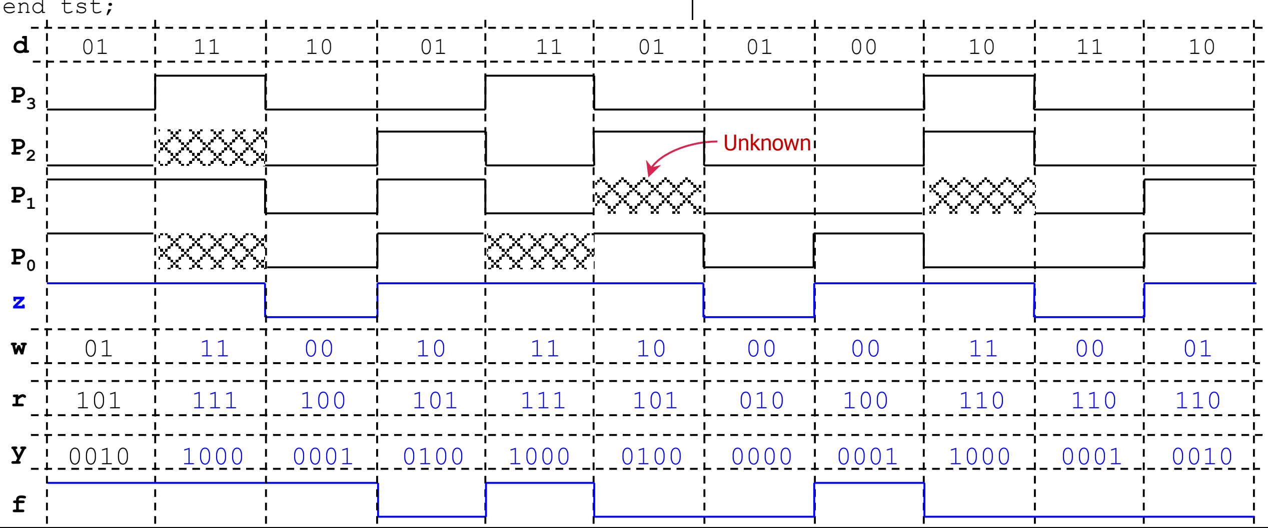
- Complete the timing diagram of the following circuit. The VHDL code (tst.vhd) corresponds to the shaded circuit.

$d = d_1d_0,\ w = w_1w_0,\ r = r_2r_1r_0,\ y = y_3y_2y_1y_0$




```
library ieee;
use ieee.std_logic_1164.all;
entity tst is
  port (d: in std_logic_vector(1 downto 0);
        r: out std_logic_vector(2 downto 0);
        u: in std_logic);
end tst;
```

```
architecture bhv of tst is

begin

  process (d, u)
  begin

     r <= d&'0';
     if u = '1' then
        r <= '1'&d;
     end if;

  end process;

end bhv;
```



PROBLEM 3 (12 PTS)

- The figure below depicts the entire memory space of a microprocessor. Each memory address occupies one byte. 1KB = 2^{10} bytes, 1MB = 2^{20} bytes, 1GB = 2^{30} bytes
  - ✓ What is the size (in bytes, KB, or MB) of the memory space? What is the address bus size of the microprocessor? (3 pts.)

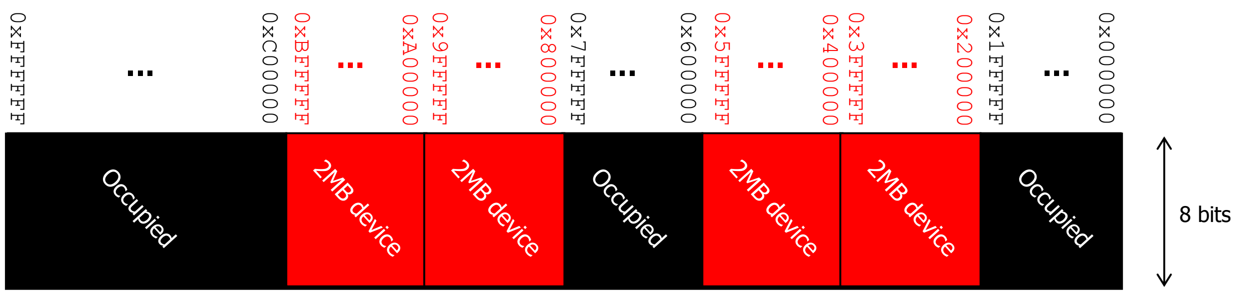
    Address space: `0x000000` to `0xFFFFFF`. To represent all these addresses, we require 24 bits. So, the address bus size of the microprocessor is 24 bits. The size of the memory space is 2^{24} = 16 MB.

  - ✓ If we have a memory chip of 2 MB, how many bits do we require to address those 2 MB of memory? (1 pt.)

    2 MB = 2^{21} bytes. Thus, we require 21 bits to address the memory device.

  - ✓ We want to connect the 2 MB memory chip to the microprocessor. For optimal implementation, we must place those 2 MB in an address range where every single address shares some MSBs. Provide a list of all the possible address ranges that the 2 MB chip can occupy. You can only use the non-occupied portions of the memory space as shown below.

    - `0x200000` to `0x3FFFFF`
    - `0x400000` to `0x5FFFFF`
    - `0x800000` to `0x9FFFFF`
    - `0xA00000` to `0xBFFFFF`



PROBLEM 4 (17 PTS)

a) Perform the following additions and subtractions of the following unsigned integers. Use the fewest number of bits n to represent both operators. <u>Indicate every carry (or borrow) from c_0 to c_n (or b_0 to b_n)</u>. For the addition, determine whether there is an overflow. For the subtraction, determine whether we need to keep borrowing from a higher bit. (6 pts.)

✓ `29 - 51`

Borrow out! → $b_6=1$, $b_5=0$, $b_4=0$, $b_3=0$, $b_2=1$, $b_1=0$, $b_0=0$

```
29 = 0x1D =  0 1 1 1 0 1 -
51 = 0x33 =  1 1 0 0 1 1
--------------------------
             1 0 1 0 1 0
```

✓ `29 + 51`

$c_6=1$, $c_5=1$, $c_4=1$, $c_3=1$, $c_2=1$, $c_1=1$, $c_0=0$

```
29 = 0x1D =  0 1 1 1 0 1 +
51 = 0x33 =  1 1 0 0 1 1
--------------------------
           1 0 1 0 0 0 0
```

Overflow! → 1 0 1 0 0 0 0

b) Perform the following operations, where numbers are represented in 2's complement. <u>Indicate every carry from c_0 to c_n</u>. For each case, use the fewest number of bits to represent the summands and the result so that overflow is avoided. (8 pts.)

✓ `29 - 51`

n = 7 bits

$c_7=0$, $c_6=0$, $c_5=1$, $c_4=1$, $c_3=1$, $c_2=0$, $c_1=1$, $c_0=0$

$c_7 \oplus c_6=0$ No Overflow

```
 29 = 0 0 1 1 1 0 1 +
-51 = 1 0 0 1 1 0 1
---------------------
-22 = 1 1 0 1 0 1 0
```

`29 - 51 = -22 ∈ [-2⁶, 2⁶-1]` → no overflow

✓ `-53 - 26`

n = 7 bits

$c_7=1$, $c_6=0$, $c_5=0$, $c_4=1$, $c_3=1$, $c_2=1$, $c_1=0$, $c_0=0$

$c_7 \oplus c_6=1$ Overflow!

```
-53 = 1 0 0 1 0 1 1 +
-26 = 1 1 0 0 1 1 0
---------------------
      0 1 1 0 0 0 1
```

`-53 -26 = -79 ∉ [-2⁶, 2⁶-1]` → overflow!

To avoid overflow: n = 8 bits (sign-extension)

$c_8=1$, $c_7=1$, $c_6=0$, $c_5=0$, $c_4=1$, $c_3=1$, $c_2=1$, $c_1=0$, $c_0=0$

$c_8 \oplus c_7=0$ No Overflow

```
-53 = 1 1 0 0 1 0 1 1 +
-26 = 1 1 1 0 0 1 1 0
-----------------------
      1 0 1 1 0 0 0 1
```

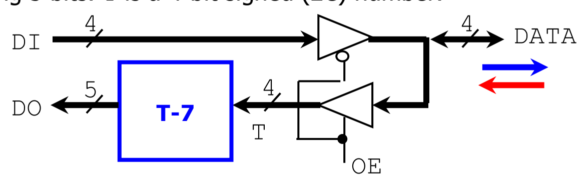
`-53 -26 = -79 ∈ [-2⁷, 2⁷-1]` → no overflow

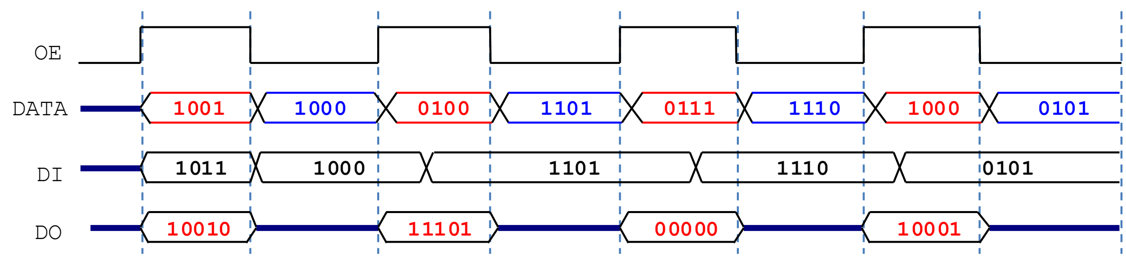
c) Get the multiplication result of the following numbers that are represented in 2's complement arithmetic with 4 bits. (3 pts.)
- ✓ -5 x 7

```
1 0 1 1 x   ➡        0 1 0 1 x
0 1 1 1              0 1 1 1
                   ---------
                     0 1 0 1
                   0 1 0 1
                 0 1 0 1
               0 0 0 0
               -------------
               0 1 0 0 0 1 1
                     ⬇
               1 0 1 1 1 0 1
```

PROBLEM 5 (11 PTS)

- Complete the timing diagram (signals DO and $DATA$) of the following circuit. The circuit in the blue box computes the signed operation T-7, with the result having 5 bits. T is a 4-bit signed (2C) number.
  - ✓ Example: if T=1010:
    DO = 1010 - 0111 = 11010 + 11001
    DO = 10011

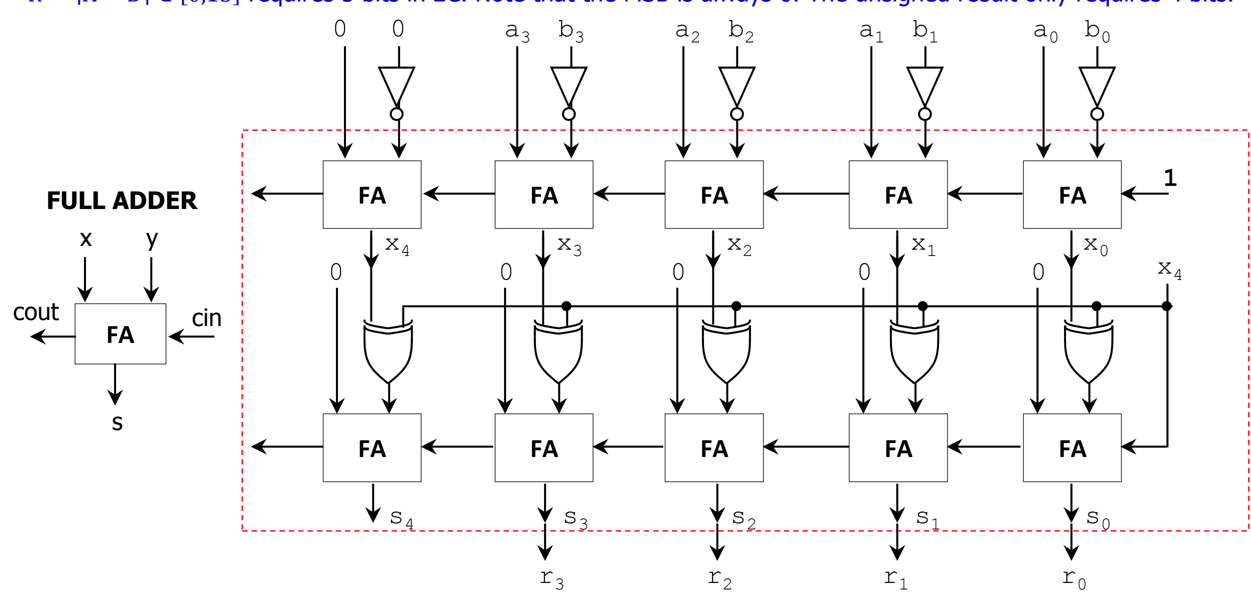




PROBLEM 6 (10 PTS)

- Sketch the circuit that computes $|A-B|$, where A, B are 4-bit unsigned numbers. For example, $A = 0101, B = 1101 \rightarrow |A-B| = |5-13| = 8$. You can only use full adders (or multi-bit adders) and logic gates. Your circuit must avoid overflow: design your circuit so that the result and intermediate operations have the proper number of bits.

$A = a_3a_2a_1a_0$, $B = b_3b_2b_1b_0$
$A, B \in [0,15] \rightarrow A, B$ require 4 bits in unsigned representation. However, to get the proper result of $A-B$, we need to use the 2C representation, where A, B require 5 bits in 2C.
- ✓ $X = A - B \in [-15,15]$ requires 5 bits in 2C. Thus, we need to zero-extend A and B to convert them to 2C representation.
- ✓ $|X| = |A-B| \in [0,15]$ requires 5 bits in 2C. Thus, the second operation $0 \pm X$ only requires 5 bits.
  - If $x_4 = 1 \rightarrow X < 0 \rightarrow$ we do $0 - X$.
  - If $x_4 = 0 \rightarrow X \geq 0 \rightarrow$ we do $0 + X$.
- ✓ $R = |A-B| \in [0,15]$ requires 5 bits in 2C. Note that the MSB is always 0. The unsigned result only requires 4 bits.



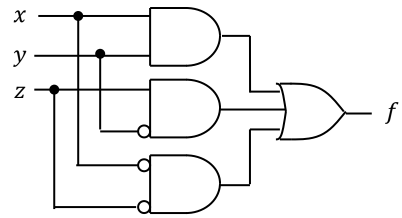
PROBLEM 7 (14 PTS)

- Given the following Boolean function: $f(x, y, z) = \prod M(3,4)$
  a) Provide the simplified expression for f and sketch this circuit using logic gates. (4 pts)

x	y	z	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

z \ xy	00	01	11	10
0	1	1	1	0
1	1	0	1	1

$f = xy + \bar{y}z + \bar{x}\bar{z}$



  b) Implement the previous circuit using ONLY 2-to-1 MUXs (AND, OR, NOT, XOR gates are not allowed). (10 pts)

$f(x, y, z) = \bar{x}f(0, y, z) + xf(1, y, z) = \bar{x}(\bar{y}z + \bar{z}) + x(y + \bar{y}z)$
$= \bar{x}g(y, z) + xh(y, z)$
$g(y, z) = \bar{y}g(0, z) + yg(1, z) = \bar{y}(1) + y(\bar{z})$
$h(y, z) = \bar{y}h(0, z) + yh(1, z) = \bar{y}(z) + y(1)$

Also: $\bar{z} = \bar{z}(1) + z(0)$